Relative full completeness for bicategorical cartesian closed structure

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Department of Computer Science

STLC = simply-typed lambda calculus with $\times$
STPC = products but no exponentials

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The classical story

STLC conservatively extends STPC

"CC-structure conservatively extends cartesian structure"

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This paper $\rightsquigarrow$ replace $\beta\eta$-equalities by witnessing isomorphisms

"up-to-isomorphism CC-structure conservatively extends up-to-isomorphism cartesian structure"

"STLC-rewriting conservatively extends STPC-rewriting"
[modulo equations]

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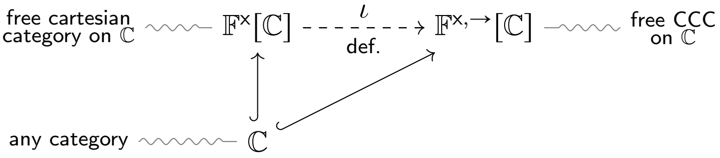
Semantic: the inclusion ι is full and faithful



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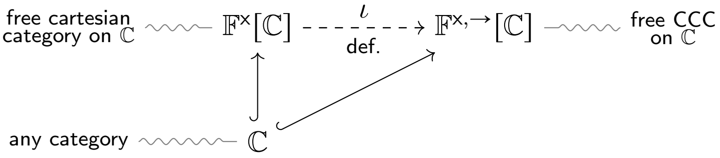

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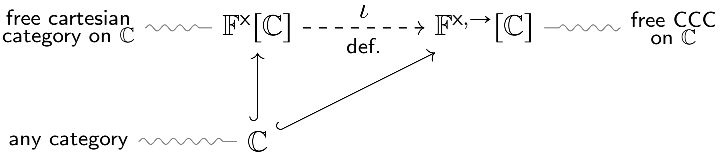

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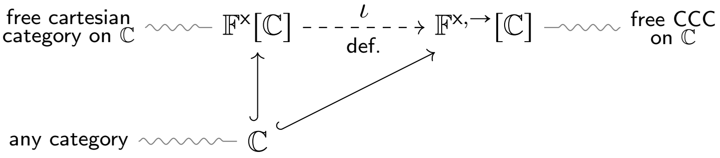

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i.e. for A, B types without $\to$,

$$\Big(\text{STPC-terms } x{:}A \vdash t{:}B\Big)\Big/_{\alpha\beta\eta} \overset{\iota_{A,B}}{\cong} \Big(\text{STLC-terms } x{:}A \vdash t{:}B\Big)\Big/_{\alpha\beta\eta}$$

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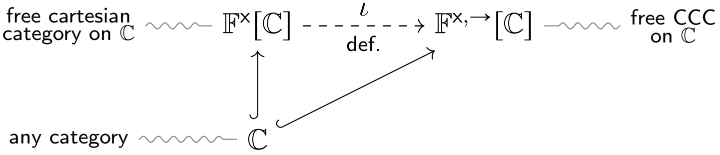

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i.e. for A, B types without $\to$,

1. Every STLC-term $t : B$ is $\beta\eta$-equal to some STPC term $t' : B$,
2. $u =_{\beta\eta} u'$ as STPC-terms iff $u =_{\beta\eta} u'$ as STLC-terms.

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How to prove it?

Syntactic: STLC is a conservative extension of STPC

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Some setup, but simple once you're there

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Ingredients:

(1) Universal property of $\mathbb{F}^{\times}[\mathbb{C}]$, $\mathbb{F}^{\times,\rightarrow}[\mathbb{C}]$,
(2) Glueing $\mathrm{gl}\,(F) = (\mathrm{id} \downarrow F)$ of a functor,
(3) Sufficient conditions for $\mathrm{gl}\,(F)$ to be a CCC,
(4) Version of Yoneda lemma,
(5) Simple facts about full / faithful functors.

STLC = simply-typed lambda calculus with $\times$
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The classical story

1. The inclusion ι is full and faithful [relative full completeness]
2. Proof is categorical [Lafont's argument]
3. Hence: STLC is conservative over STPC.

Proof principle:

can prove facts about STPC / cartesian categories
using STLC / CCCs.

The bicategorical story

Bicategorical cartesian closed structure

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objects $A, B, \ldots$
1-cells $f, g : A \to B$
2-cells $\tau, \sigma : f \Rightarrow g : A \to B$

$(f \circ g) \circ h \cong f \circ (g \circ h)$
$f \circ \mathrm{Id} \cong f \cong \mathrm{Id} \circ f$

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$$\pi_i\langle f_1, f_2\rangle = f_i \qquad f = \langle \pi_1 \circ f, \pi_2 \circ f\rangle$$
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$\beta\eta$-equalities ⇝ witnessing isomorphisms

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Cartesian closed structure

bicategorical models of STLC

[Seely, Hilken, Hirschowitz, Olimpieri,. . .]

bicategorical models of linear logic

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Question: does relative full completeness hold for bicategories?
Answer: yes!

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Question: does relative full completeness hold for bicategories?
Answer: yes! $\rightsquigarrow$ but what about the syntactic side?

Bicategorical STPC and STLC [Fiore & S., LICS'19]

cartesian categories = models of STPC

cartesian closed categories = models of STLC

Bicategorical STPC and STLC [Fiore & S., LICS'19]

finite product bicategories = models of $\Lambda_{\text{ps}}^{\times}$

cartesian closed bicategories = models of $\Lambda_{\text{ps}}^{\times,\rightarrow}$

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Judgements

$\Gamma \vdash t : A$ [1-cells]

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$\Gamma \vdash \tau \equiv \sigma : t \Rightarrow t' : A$

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$$\Gamma \vdash \pi_i \{\langle t_1, t_2 \rangle\} \stackrel{\beta_i}{\Longrightarrow} t_i : A_i$$

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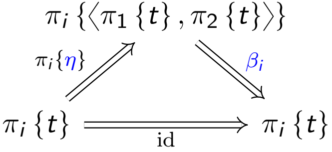

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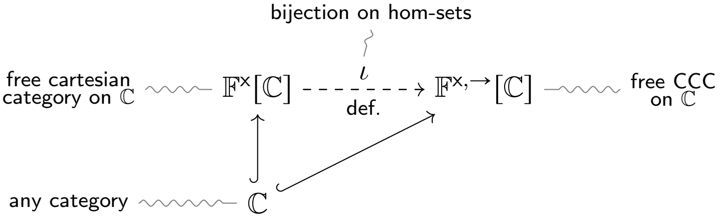
Rewrites describe $\beta\eta$-rewriting [modulo equations]

where $t \simeq^A_B t'$ iff there exists a rewrite $x : A \vdash \tau : t \Rightarrow t' : B$:

$$\left(\Lambda^{\times,\to}_{\mathrm{ps}}\text{-terms } x{:}A \vdash t{:}B\right)\Big/_{\alpha\simeq^A_B} \cong \left(\text{STLC-terms } x{:}A \vdash t{:}B\right)\Big/_{\alpha\beta\eta}$$

Relative full completeness for categorical CC-structure



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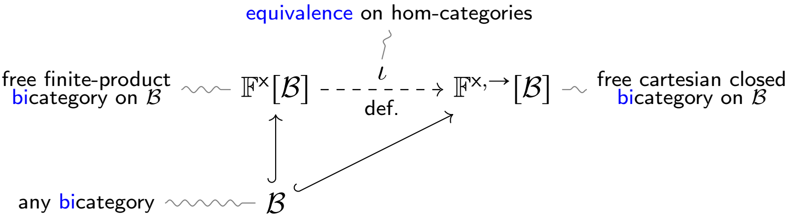

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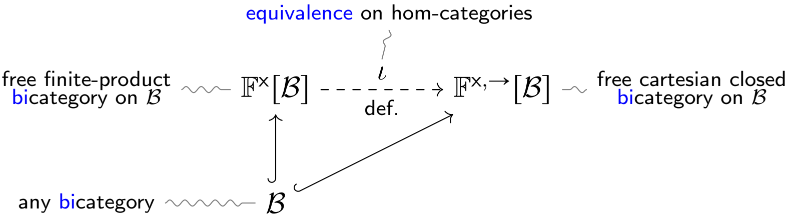

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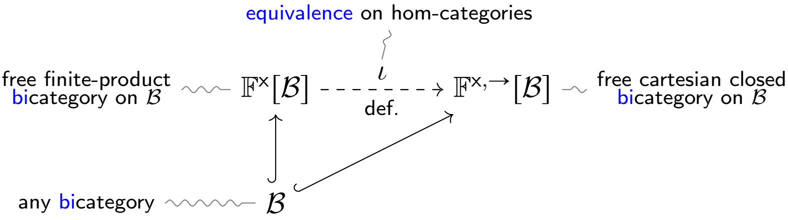

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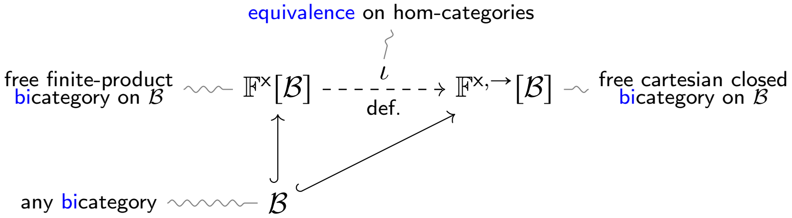

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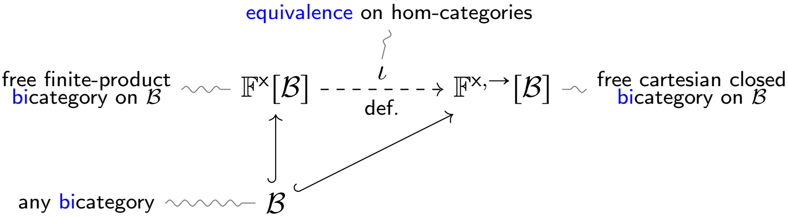
i.e. "conservativity of 2-cells": for A, B without $\to$

$$\big(\text{2-cells } \sigma : t \Rightarrow t' : A \to B \text{ built with product structure}\big)$$
$$\text{|||}\wr$$
$$\big(\text{2-cells } \sigma : t \Rightarrow t' : A \to B \text{ built with CC-structure}\big)$$

Relative full completeness for bicategorical CC-structure



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i.e. "conservativity of rewriting": for A, B without $\rightarrow$

$$\left(\Lambda^{\times}_{\mathrm{ps}}\text{-rewrites } x{:}A \vdash \tau{:}t \Rightarrow t'{:}B\right)\big/_{\equiv} \;\cong\; \left(\Lambda^{\times,\rightarrow}_{\mathrm{ps}}\text{-rewrites } x{:}A \vdash \tau{:}t \Rightarrow t'{:}B\right)\big/_{\equiv}$$

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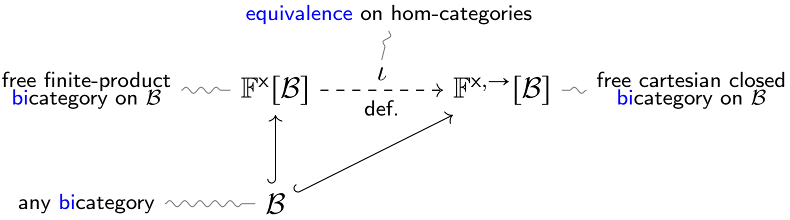

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i.e. "conservativity of rewriting": for A, B without $\rightarrow$

1. Every $\Lambda_{\mathrm{ps}}^{\times,\rightarrow}$-rewrite $\tau : t \Rightarrow t'$ is $\equiv$-equal to some $\Lambda_{\mathrm{ps}}^{\times}$-rewrite,
2. $\sigma \equiv \sigma'$ as $\Lambda_{\mathrm{ps}}^{\times}$-rewrites iff $\sigma \equiv \sigma'$ as $\Lambda_{\mathrm{ps}}^{\times,\rightarrow}$-rewrites.

STLC = simply-typed lambda calculus with $\times$
STPC = products but no exponentials

This paper: the bicategorical story

1. The inclusion ι is locally an equivalence [relative full completeness]
2. Proof is bicategorical [Lafont's argument]
3. Hence: STLC-rewriting is conservative over STPC-rewriting
   [modulo equations] [modulo equations]

Proof principle:

can prove facts about STPC-rewriting / fp-bicategories
using STLC-rewriting / cartesian closed bicategories.

What's in the paper?

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1. *The glueing category $\mathrm{gl}\,(F)$ is cartesian closed,*
2. *The forgetful functor $U : \mathrm{gl}\,(F) \to \mathbb{C}$ strictly preserves cartesian closed structure.*

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   [defined as a comma bicategory $\mathrm{id} \downarrow F$]
2. Prove "fundamental lemma",
3. Show $\iota : \mathbb{F}^{\times}[\mathcal{B}] \hookrightarrow \mathbb{F}^{\times,\to}[\mathcal{B}]$ is locally an equivalence.
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Type-theoretic interpretation

if t, t' in STPC and $\tau : t \Rightarrow t'$ constructed using CC-structure, there exists $\sigma : t \Rightarrow t'$ such that

1. $\sigma \equiv \tau$,
2. σ constructed using finite product structure.